# Final Project Problems

On Friday, April 18 and April 25, we will not have a formal class. Instead you will work ***one*** of the problems listed below and turn in a 1-2 page, single spaced report as described below. You must write a program in FORTRAN or C (C++) to obtain your solution. Your project is due in the green homework box in the Physics Department main office by 5 p.m. on Friday, May 2. No work submitted after this date will be graded.

This final project is worth 20% of your overall course grade and will be graded on a 10 point scale. The table outlines what you should turn in.

| What to Submit | Point Value |
|---|---|
| 1-2 page report | 5 |
| graph(s) of your solution(s) | 3 |
| copy of your FORTRAN or C/C++ program | 2 |

Your report should include the following information:

| What Report Should Include |
|---|
| **Introduction:** Description of problem and what you will solve for |
| **Solution Method:** Explanation of your method of solution. Explain how you designed your program and what various subroutines do. Specify which algorithms you used to perform calculations. |
| **Checks on Solution:** Explanation of the methods you used to check your solution. Explain how the result verified what you expected based on your knowledge of physics. |
| **Results and Discussion:** Describe your numerical results and graphs in words and discuss their physical significance. Address the questions asked in the problems. |
| **Conclusions:** State what you conclude from your solution of the problem. |

If you have any questions or difficulties, we will be available in I E 124 (our usual meeting place for programming) from 2 to 5 p.m. on Friday, April 18 and April 25, to answer questions.

## Notes:

1. Make sure you label the graphs (particularly the axes and data series) in a meaningful way.

## Final Project Problems (Pick One)

**(1) Olympic Long Jump:** During the 1968 Olympic Games in Mexico City, R. Beamon of the United States set a world record of 29 feet 2.5 inches in the long jump, improving the previous world record by over 2.5 feet. Many critics attributed this to the thinness of the air in Mexico City. If one writes Newton's Second Law assuming the air resistance is proportional to the ***square*** of the jumper's velocity, one gets the following equations:

$$m\frac{dv_x}{dt} = - \ kv_x\sqrt{v_x^2 + v_y^2}$$

$$m\frac{dv_y}{dt} = -mg - \ kv_y\sqrt{v_x^2 + v_y^2}$$

where $v_x$ and $v_y$ are the $x$ and $y$ components of the jumper's velocity, is the air density, $k$ is the drag constant, and $m$ is the jumper's mass. Solve the above coupled equations numerically for the following **initial conditions** and **parameter values**: $v_{x0}$=9.45 m/s, $v_{y0}$=4.15 m/s, $k$=0.182 m$^2$, =0.984 kg/m$^3$ (for Mexico City), $m$=80 kg (approximate mass of R. Beamon).

(1) Plot $v_x$ and $v_y$ versus time $t$. Does the program give you what you expect for the case where $k$ is large?

(2) Plot the trajectory of the jumper ($y$ versus $x$.)

(Hint: recall that $x(t) = \int_0^t v_x dt$ and $y(t) = \int_0^t v_y dt$ )

(3) Study the trajectory as a function of the air density . If the density doubles by what fraction does the range of the jump change in this model, assuming a level field?

(4) Study the trajectory as a function of drag constant $k$. If $k$ doubles, by what fraction does the range of the jump change? What conclusions may be drawn from this model concerning the effects of air resistance on the range of the jump?

**(2) <u>Quantum Mechanics Problem</u> :** The equation which describes the quantum mechanical behavior of a single non-relativistic particle in one dimension is

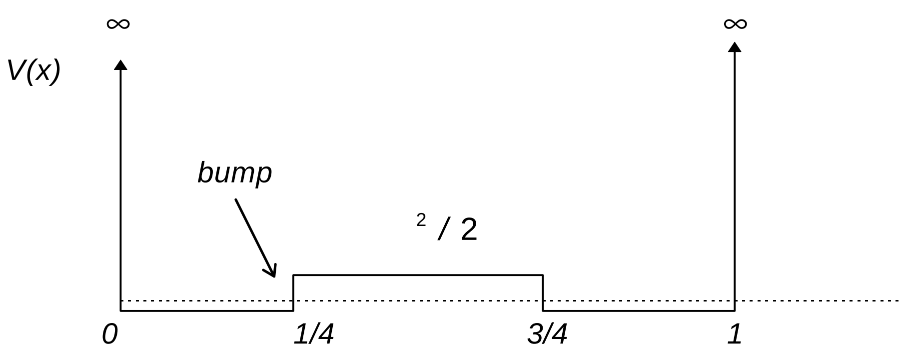
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \quad = E \qquad \text{where } \hbar \equiv \frac{h}{2}$$

(Planck's constant divided by 2π), $V(x) \equiv$ (the potential energy of the particle as a function of position), $\equiv$ the wave function of the particle, $E$ is the energy of the particle, and m is the mass of the particle. We assume the mass of the particle is such that $\frac{\hbar^2}{2m} = 1$, giving the equation

$$-\frac{d^2}{dx^2} + V(x) \quad = E$$

Suppose that the particle moves in a one dimensional potential given by

$$V(x) = \begin{cases} \infty & \text{for } x \le 0 \\ 0 & \text{for } 0 < x < 1/4 \\ {}^2/2 & \text{for } 1/4 < x < 3/4 \\ 0 & \text{for } 3/4 < x < 1 \\ \infty & \text{for } 1 < x \end{cases}$$




(1) Find the ground state energy (lowest non-zero energy) and wave function of the particle assuming the above potential. Note that without the "bump" in the middle, the problem is identical to that of vibrations in a wire with fixed ends, and the ground state is $E_0 = \frac{\hbar^2 k^2}{2m} = k^2 = {}^2$. Do you expect the bump to increase or decrease the ground state energy? Graph the ground state wave function.

(2) Calculate the expectation value of x for the ground state wave function, i.e. the average position of the particle. The expectation value $\langle x \rangle$ for this case is

$$\langle x \rangle = \frac{\int_0^1 {}_0^2(x)x\,dx}{\int_0^1 {}_0^2(x)\,dx}$$

(3) Calculate and graph the ground state wave function for a "very large" bump. Does the result agree with what you expect?